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Robust Design Approach for Achieving Flexibility in Multidisciplinary Design

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The interdisciplinary nature of complex systems design presents challenges associated with computational burdens and organizational barriers as these issues cannot be resolved with faster computers and more efficient optimization algorithms. There is a need to develop design methods that can model different degrees of collaboration and help resolve the conflicts between different disciplines. An approach to providing flexibility in resolving the conflicts between the interests of multiple disciplines is proposed. We propose to integrate the robust design concept into game theory protocols, in particular the Stackelberg leader/follower protocol. Specifically, the solution for the design parameters that involve the coupled information between multiple players (disciplines) is developed as a range of solutions rather than a single point solution. This additional flexibility provides more freedom to the discipline that takes the role of follower, while also keeping the performance of the leader discipline stable within a tolerable range. The method is demonstrated by a passenger aircraft design problem.

Nomenclature

B = wing span, ft
f_i = objective functions of discipline i
g_i = constraints of discipline i
k_j = penalty factors in robust design constraints
L = fuselage length, ft
Ld_c = lift-to-drag ratio on climb
Ld_l = lift-to-drag ratio on landing
Ld_t = lift-to-drag ratio on takeoff
R_{fr} = fuel ratio required
S = wing area, ft^2
Ti = installed thrust, lb
V_{br} = best-range velocity, ft/s
Wto = takeoff weight, lb
X_i = design variables for each discipline i
x_L = lower bound of design variables
x_U = upper bound of design variables
y_{ij} = linking variables that are evaluated by discipline i and required by discipline j as the input
Δx = deviation range of design solution
μ_f = mean of the objective function f
σ_f = standard deviation of the objective function f

I. Introduction

THE interdisciplinary nature of modern design processes presents additional challenges beyond those encountered in designs that only involve a single discipline. It increases computational burdens because of the complexity of the problem and also creates organizational challenges for implementing the necessary cross-disciplinary couplings. Several design architectures have been developed to support collaborative multidisciplinary design environments using distributed design optimizations. The concurrent subspace optimization (CSSO) approach[1–3] and the collaborative optimization (CO) approach[4–6] are the two main streams of work in this area. A comprehensive review of the multidisciplinary design optimization architectures is provided by Kroo[7] and will not be repeated here.

Although concurrent multidisciplinary optimization, including the all-at-one approach, represents an ideal way of addressing simultaneously the needs of multiple disciplines, the issues of computational burdens and organizational challenges likely cannot be resolved with faster computers and more efficient optimization algorithms. A total cooperation among disciplines in a concurrent engineering environment is rare in practice because of the aforementioned reasons. Researchers are working on developing the mathematical constructs that could model degrees of collaboration and allow subdisciplines to make decisions independent of the decisions of others. One of these approaches is the game theoretic approach. Lewis and Mistree[8] model multidisciplinary optimization problems using game theoretical principles, which are rooted in decision science. In their model design processes are abstracted as games, and the disciplinary design teams and their associated analysis/synthesis tools are the players in the game. Different degrees of collaboration can be characterized by game protocols, and the details are provided later in this paper.

In this work we propose to integrate the robust design concept with game-theory protocols to provide flexibility in multidisciplinary decision making. Specifically, our approach is applied to the Stackelberg leader/follower formulation[8,9] in which decisions of subdisciplines are not made completely concurrently but occur sequentially. Note that the Stackelberg leader/follower approach is different from the traditional sequential approach to design. In the sequential approach the initial decisions are made without any formal consideration of the later disciplines. Any information necessary up front is just ignored or assumed. In the Stackelberg approach the leader assumes rationality of the followers. Although this assumption seems subtle, it is important, as will be seen later in this paper.

Our aim is to provide flexibility in a design process and help to further resolve the conflicts and disputes of rationality between the interests of multiple disciplines. By flexibility, we mean that instead of looking for a single point solution in one discipline's model we look for a range of solutions that involve information passing between multiple players (disciplines). With this flexibility the design freedom of individual disciplines, especially the one that takes the follower's role, could be significantly improved. Ultimately, this process will result in better products in less time because fewer iterations are needed, more flexibility is allowed, and disciplinary decisions are made considering the actions of other disciplines.

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II. Technical Background

A. Game Protocols for Multidisciplinary Optimization

In a typical multidisciplinary optimization problem the information flow between multiple disciplines could be represented by Fig. 1.

In Fig. 1, X_i are the design variables for each discipline; y_{ij} are the linking variables that are evaluated in the analysis of discipline i and required by discipline j as the input of its analysis; f_i are the objective functions; and g_i are the constraints. The linking variables y_{ij} could include some of the design variables X_i; there may also exist overlaps of objectives and constraints among the disciplines, and a part of the design variables X_i may be shared by different disciplines.

It was recognized in Ref. 10 that game theory can be used to model the preceding scenario where the disciplines are treated as players in the game of design. In Ref. 11 the fundamental constructs of three protocols applicable to design are developed. These are cooperative, noncooperative, and sequential decision-making scenarios. A short conceptual discussion of each protocol is given here. More mathematical details of the protocols are given in Refs. 8, 9, and 12.

Cooperative: Complete cooperation occurs when each designer is aware of all of the others and the decisions made by each. In mature design problems where complete information is available and the transfer of information is close to if not seamless, the assumption of perfect or approximate communication is extremely beneficial.[1] Cooperative solutions, or Pareto solutions, are solutions where both players cannot simultaneously improve.[13] These are desirable solutions.

Noncooperative: Design teams may not have the necessary information they need to make a decision. Each design team will have to make assumptions, many times worst case, about the information needed from other teams because of interpersonal, computational, or organizational isolation. This scenario is known as a Nash noncooperative formulation.[14]

Sequential (Stackelberg leader/follower): Leader/follower relationship exists among design teams where one team makes their decision or finalizes their design and passes this information onto the next team. This relationship, although not fully cooperative, does have some information transfer. The leader, or most dominant design team must assume something about the behavior of the teams following it, and the follower gets to use the information from the preceding teams but is also constrained by it.

Whereas the complete cooperation is the most desired situation, the leader/follower relationship occurs in many cases. As an example, in aircraft design, typically the propulsion systems design is conducted before other disciplines such as structures, controls, etc. However, the controls or structures designers could create a design that renders the propulsion design useless or not powerful enough. This would create time-consuming iterations. Moreover, it may be even impossible for the follower to find a feasible design given the design parameters assigned by the leader. Therefore the typical sequential design approach may result in conflicts because decisions about variables that influence multiple disciplines are made by one of the several involved parties based on the incomplete information.

Of interest in this work is the integration of the robust design concept into the Stackelberg leader/follower model. The Stackelberg leader/follower formulation is different from a typical sequential formulation in that it uses a concept known as the rational reaction set (RRS) that strengthens the link between the leader and follower. Originating in game theory, the term RRS here implies a set of solutions that an isolated decision maker constructs as a function of unknown information from other decision makers. In the leader/follower formulation the leader knows the RRS of the follower. In other words the leader knows how the follower will react to the decisions the leader makes. The rationale is that if the leader can predict to some extent how the follower will react or even account for the follower's interest in their design the system design will be more effective, and needless iterations will be avoided.

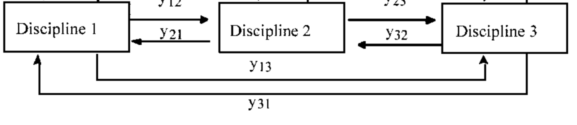

Fig. 1 Information flow of a multidisciplinary system.

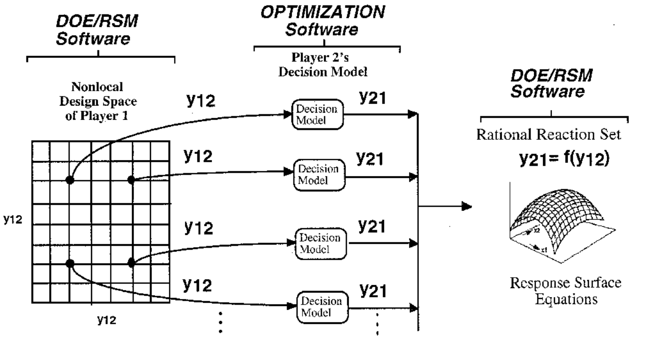

Fig. 2 Construction of rational reaction sets.

In Lewis's work[8] the RRS is constructed based on an experimental approach using samples in a nonlocal design space. Design of experiments (DOE) techniques[15] are used to sample different design points from one player. These points are then fed to another player's model, and the model is solved using optimization software. In this work the optimization software used is Decision Support in the Design of Engineering Systems (DSIDES).[16] This is repeated a number of times, depending upon the number of coupled variables and the level of the response surface desired. A response surface is created linking the solution of one player as a function of the solution of another using DOE techniques. This process is illustrated in Fig. 2, where the rational reaction set of player 2 is constructed $[y_{21} = f(y_{12})]$. Points are sampled in player 1's space (y_{12}), and then these values are fed to player 2's decision model, which is solved (y_{21}). Then the input/output pairs, where y_{12} represents the input and y_{21} represents the output, are used to construct the response surface approximation of $y_{21} = f(y_{12})$ (right side). This RRS will then be used in P^1's model as a prediction of P^2's behavior.

Although the RRS provides a viable approach to addressing the needs of the follower in the leader's model, the results obtained from the leader/follower model may deviate from those obtained using the all-in-one, or cooperative, approach. In this work we propose to further resolve the conflicts between multiple players using the robust design method. In the next section we introduce the concept of robust design and its usefulness in achieving flexibility. Robust design techniques are used to help alleviate some of the conflicts in the leader/follower approach.

B. Robust Design Approach to Achieving Design Flexibility

Taguchi's robust design method has been widely used to design quality into products and processes.[17–19] Whereas various other approaches assume that a good design meets a set of well-defined functional, technical performance, and cost goals, Taguchi states that a good design minimizes the quality loss over the life of the design, where quality loss is defined to be the deviation from the desired performance.

Whereas the majority of the early applications of robust design consider manufacturing as the cause for performance variations, recent developments in design methodology have produced design approaches and methods that introduce the robustness of design decisions.[20,21] In Chang' work Taguchi's parameter design concept is used to support teams in communicating about sets of possibilities and to make decisions that are robust against variations in the part of the designs done by other team members. In their model the uncertainties between different teams are modeled as noise factors (uncontrolled parameters). A part of the author's investigation[21] is to apply the robust design concept to the early stages of design for making decisions that are robust to the changes of downstream design considerations (called type I robust design). Furthermore, the robust design concept is extended to make decisions that are flexible to be allowed to vary within a range (called type II robust design). What is relevant to this work is the type II robust

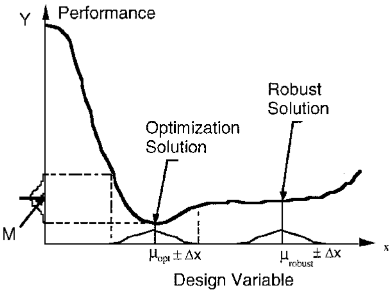

Fig. 3 Type II robust design: developing flexible solutions.[21]

design, in which performance variations are contributed by the deviations of control factors (decision variables) rather than the noise factors.

The concept behind type II robust design for determining flexible design solutions is represented in Fig. 3. For purposes of illustration, assume that the performance is a function of only one variable x. Generally, in this type of robust design, to reduce the variation of response caused by variations of design variables, instead of seeking the optimum value, a designer is interested in identifying the flat part of a curve near the performance target. If the objective is to move the performance function toward target M and if a robust design is not sought, then obviously the point $x = \mu_{\text{opt}}$ is chosen. However, for a robust design $x = \mu_{\text{robust}}$ is a better choice. If design variables vary within the region $\pm\Delta x$ of their means, the resulting variation of response of the design at $x = \mu_{\text{robust}}$ is much smaller than that at $x = \mu_{\text{opt}}$, while the means of the response at two designs are close.

When implemented by optimization, robust design is achieved by bringing the mean on target (aligning the peak of the bell-shaped response distribution with the targeted quality) and minimizing the variance (making the bell-shaped curve thinner). For a typical optimization model that is stated in Eq. (1),

Find:

$$x$$

Minimize:

$$f(x) \tag{1}$$

Subject to:

$$g_j(x) \leq 0, \qquad j = 1, 2, \ldots, J$$

$$x_L \leq x \leq x_U$$

The robust optimization can be formulated as a multiobjective optimization problem shown as the following:

Find:

$$x, \Delta x$$

Minimize:

$$[\mu_f, \sigma_f] \tag{2}$$

Subject to:

$$g_j(x) + k_j \sum_{i=1}^{n} \left| \frac{\partial g_j}{\partial x_i} \right| \Delta x_i \leq 0, \qquad j = 1, 2, \ldots, J$$

$$x_L + \Delta x \leq x \leq x_U - \Delta x$$

where μ_f and σ_f are the mean and the standard deviation of the objective function $f(x)$, respectively. In Eq. (2), the mean locations and the range of design solutions are identified as x and Δx. To study the variation of constraints, we use the worst-case scenario, which assumes that all variations of system performance may occur simultaneously in the worst possible combination of design variables.[17] To ensure the feasibility of the constraints under the deviations of the design variables, the original constraints are modified by adding the penalty term to each of them, where k_j are penalty factors to be determined by the designer. The bounds of design variables are also modified to ensure the feasibility under deviations. Depending on the computation resource, μ_f and σ_f could be obtained through simulations or analytical means such as Taylor expansions. The robust design approach introduced in this section is applied to multidisciplinary optimization to improve the flexibility of a decision-making procedure.

III. Approach for Achieving Flexibility in Multidisciplinary Optimization

To facilitate the following discussion, assume there are two players P^1 and P^2 that control sets of x_1 and x_2 and try to minimize their own sets of objective functions f_1 and f_2, respectively. If we take P^1 as a leader and P^2 as a follower, based on the nomenclature provided in Fig. 1, the leader/follower model developed by Lewis[11] is illustrated in Fig. 4.

In the preceding model the leader P^1 makes decisions first based on the rationality of the follower. This rationality is captured by the RRS of the follower, represented by the relationships between linking variables, i.e., $y_{21} = \text{RRS}(y_{12})$, through response surface approximations as introduced in Sec. II.A. The leader's solution, specifically the linking variable y_{12}, will then be passed to the follower's model so that player 2 could make decisions accordingly. Although RRS provides a viable approach to addressing the needs of the follower in the leader's model, the follower may still not be able to find a favorable design as it is constrained by the leader.

To resolve this conflict, the robust design formulation introduced in Sec. II.B is incorporated to increase the freedom of the follower. As shown in Fig. 5, different from the model in Fig. 4, this new approach will generate a range of solutions $[x_1 - \Delta x_1, x_1 + \Delta x_1]$ instead of a single point solution in the leader's model. This range of solutions will determine the performance of P^1, which are good enough, and at the same time will be allowed to vary in an acceptable range. Given the range of x_1 and the resulting range of linking variables y_{12}, it follows naturally that it is easier for P^2 to find a favorable design. As shown in Fig. 5, in the Find section of P^2's model, the follower will determine its own best solution x_2 and at the same time is given the opportunity to pick the most favorable value of linking variables y_{12} passed from the leader. In this way the follower's performance can be improved, and the leader's performance is still stable in the range although it may not be the best. The

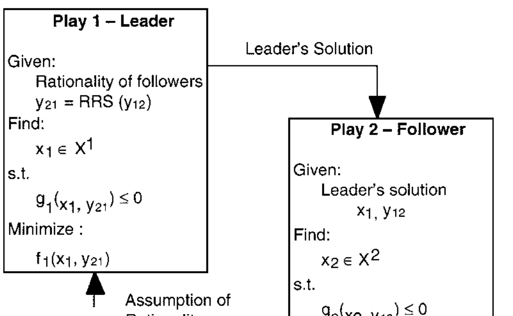

Fig. 4 Leader/follower protocol models.

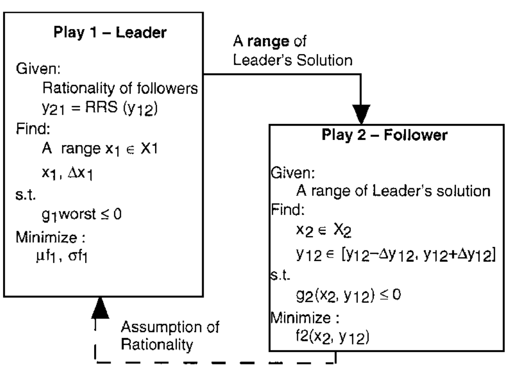

Fig. 5 Leader/follower model with robustness considerations.

overall solution should be at least comparable to that of the model without robust design considerations. In Fig. 5, $g_{1\text{worst}}$, the worst value of the constraint g_1, can be derived following the discussions provided earlier for Eq. (2). A range of solutions could be developed for the whole set or only a part of leader's design variables. For those design variables x_1 that show no effect on the linking variables y_{12} (they ultimately have no impact on follower's solution), a single point solution could be sought.

Because robust design involves multiple objectives such as the optimization of mean performance μ_f and the minimization of the performance variance σ_f [Eq. (2)], multiobjective optimization techniques need to be employed to support the tradeoffs of these two aspects. There are many existing multiobjective optimization approaches that can be used to achieve this purpose. In this work the compromise decision support problem (DSP) is selected to implement the multiobjective robust design optimization. The compromise DSP (shown in Fig. 6) is a multiobjective mathematical construct, which is a hybrid formulation based on mathematical programming and goal programming.[22] In the compromise DSP each goal has two associated deviation variables d_i^- and d_i^+ that indicate the extent of the deviation from the target G_i. Goals may either be weighted in an Archimedean solution scheme or, using a preemptive approach, rank ordered into priority levels to effect a solution on the basis of preference.[22] The goals are transformed into a total deviation function, a formula comprised of deviations variables, as indicated by Z in Fig. 6. Using the compromise DSP, a robust design problem is modeled with two separate goals for bringing the mean on target and minimizing the deviation of the system performance. Each player in Fig. 5 has their own compromise DSPs. However, the leader's includes the robust formulation.

Find
Design Variables X_i $i = 1, \ldots, n$
Deviation Variables d_i^-, d_i^+ $i = 1, \ldots, m$
Satisfy
System constraints (linear, nonlinear)
System goals (linear, nonlinear)
$$A_i(\mathbf{X}) + d_i^- - d_i^+ = G_i;\ i = 1, \ldots, m$$
Bounds
$$X_i^{min} \le X_i \le X_i^{max};\ i = 1, \ldots, n$$
$$d_i^-, d_i^+ \ge 0;\quad i = 1, \ldots, m$$
$$d_i^- \cdot d_i^+ = 0;\quad i = 1, \ldots, m$$
Minimize
Preemptive deviation function (lexicographic minimum)
$$\mathbf{Z} = [f_1(d_i^-, d_i^+), \ldots, f_k(d_i^-, d_i^+)]$$

Fig. 6 Compromise decision support problem.

IV. Example: Design of a Passenger Aircraft

To illustrate the effectiveness of the approach, the design of a 727(200 passenger aircraft), which is derived from Refs. 8, 11, and 23, is investigated. Two distinct subsystems are identified, each with their own analysis and synthesis routines: the aerodynamics subsystem is responsible for the wing and fuselage lift characteristics, and the weights subsystem is responsible for setting the thrust available and takeoff weight through a fuel balance. In Fig. 7 the original compromise DSPs for each player, before the RRS equations and robust design considerations are introduced into the model, are shown.

The relationship between these two subsystems is further illustrated in the dependency diagram Fig. 8, where B is the wing span, S is the wing area, L is the fuselage length, Wto is the takeoff weight, and Ti is the installed thrust. There are three linking variables, y_{21}—Wto, Ti, and Rfr (fuel balance)—required by the aerodynamics designer from the weights designer, and five linking variables y_{12}—S, Vbr (best range speed), and three lift-to-drag ratios Ld_c, Ld_t, and Ld_l—required by the weights designer from the aerodynamics designer. Both sets of linking variables include a part of the design variables. The details of the two disciplinary models are given in Refs. 11 and 23. The models used are sets of analytical equations. However, the solution of each discipline's model not only includes optimization iterations, but also includes analysis iterations where each discipline must find an equilibrium point. For instance, the weight discipline must perform a fuel balance, which requires iterating and bringing the fuel required and fuel available into equilibrium. Also, the aerodynamics discipline must bring the drag coefficients and velocities into equilibrium with each analysis.

The procedure of constructing the RRS between the two disciplines is introduced in Ref. 11 and described briefly in Sec. II.A, and will not be repeated here. The RRS of each discipline, approximating the linking variables as a function of the other player's linking variables, are provided in the Appendix. All of these second-order response surface approximations are used in each scenario to predict the influence of one discipline upon another.

Our approach is illustrated for the case in which the aerodynamics (Aero) system is the leader and the weights (Weights) system

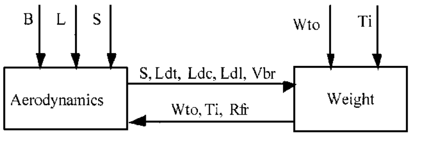

Fig. 8 Dependency diagram of passenger aircraft model.

AERODYNAMICS

Find
The values of the **aerodynamics variables**

		Units
Wing area,	S	[ft²]
Fuselage length,	l	[ft]
Wing span,	b	[ft]

The values of the **deviation variables** associated with the goals
Satisfy
The **aerodynamics constraints**
The aspect ratio must be less than 10.5
The achievable climb gradient on landing must be ≥ 2.4°
The achievable climb gradient on landing must be ≥ 2.7°
The landing field length must be ≤ 4,500 ft.
The take-off field length must be ≤ 6,500 ft.
The drag coefficient in take-off and landing must be ≥ 0.02
The drag coefficient in cruise must be ≥ 0.02
The **aerodynamics goals**
Missed Approach Climb Gradient, landing
Missed Approach Climb Gradient, take-off
Landing Field Length
Take-off Field Length
Aspect Ratio
The **bounds** on the **aerodynamics variables**
Minimize
The sum of the **deviation variables**
$$Z = \Sigma W_i(d_i^- + d_i^+)$$

WEIGHTS

Find
The values of the **weights variables**

		Units
Take-off Weight,	Wto	[lbs]
Installed Thrust,	Ti	[lbs]

The values of the **deviation variables** associated with the goals
Satisfy
The **weight constraints**
The useful load must be less than 0.3
The fuel available must be ≥ the fuel required
The achievable climb gradient on landing must be ≥ 2.4°
The achievable climb gradient on landing must be ≥ 2.7°
The landing field length must be ≤ 4,500 ft.
The take-off field length must be ≤ 6,500 ft.
The **weights goals**
Productivity Index
Useful Load Fraction
Fuel Balance
Missed Approach Climb Gradient, landing
Missed Approach Climb Gradient, take-off
Landing Field Length
Take-off Field Length
The **bounds** on the **weights variables**
Minimize
The sum of the **deviation variables**
$$Z = \Sigma W_i(d_i^- + d_i^+))$$

Fig. 7 Aerodynamics and Weights compromise DSPs.

Given

Relevant constants
Important Aerodynamic Relationships and Equations
Rational Reaction Sets of Player Weight's linking variables Wto, Ti, R_{fr} as functions of S, Ld_c, Ld_t, Ld_l, and V_{br} (Player Aero's variables)

Find

Aerodynamics Design Variables L, B, S
Most favorable linking variables S, Ld_c, Ld_t, Ld_l, and V_{br}
Deviation Variables associated with Aero goals, d_i^-, d_i^+

Satisfy

Seven (7) aerodynamic constraints
Five (5) goals on aerodynamic mean performance
Five (5) goals on aerodynamic performance variance
Variable Bounds (upper and lower bounds)

Minimize Deviation Function

$$Z = [f_1(d_i^-, d_i^+), ..., f_k(d_i^-, d_i^+)]$$

Fig. 9 Robust design model for Aero as leader.

Given

Relevant constants
Important Fuel/Weights Relationships and Equations
Solution of Aerodynamics S, ΔS, Ldc, Ldt, Ldl, Vbr

Find

Weights Design Variables Wto, Ti
Most favorable value of linking variable S within ΔS

Satisfy

Weights constraints (6 nonlinear constraints)
Weights goals (7 nonlinear goals)
Variable Bounds (upper and lower bounds)

Minimize Deviation Function

$$Z = [f_1(d_i^-, d_i^+), ..., f_k(d_i^-, d_i^+)]$$

Fig. 10 Model for player Weights as follower.

Table 1 Solutions of Aero leader/Weights follower models without robust design considerations

Parameter	Aero (leader)	Weights (follower)
Design variables	*B*, ft; 138.377	*Ti*, lb; 30,269.2
	S, ft^2; 1,913.04	*Wto*, lb; 188,119
	L, ft; 136.418	—— ——
Deviation function of performance	0.2402	0.2130

is the follower. Without including the robust design considerations, the problem can be solved sequentially by the Stackelberg leader/follower models as presented in Fig. 5. The results of design variables and the achieved performance from the nonrobust leader/follower formulation are presented in Table 1. These results will be used as benchmarks for the robust formulations.

To introduce the flexibility into the leader/follower decision making process, the Aero design model is first modified based on the robust design principle (Fig. 9). A range of solutions is sought for one design variable S (wing area), which is also a linking variable (y_{12}) passing to the Weights. The range of wing area ΔS is thus considered as an additional design variable in the Aero leader's robust design model. In addition to satisfying the constraints in the worst case of design deviations, the design objectives are modified to accommodate the goal of minimizing the performance as well as optimizing the mean performance. All of the design objectives are captured as goals in the compromise DSP.

Three different design scenarios are considered for solving the preceding model. As shown in Table 2, for the first two scenarios the preemptive formulation is used where the robust and performance goals are placed at different priority levels. In scenario III the Archimedean formulation is used in which the aforementioned two sets of goals are placed at the same level with equal weights. To evaluate the mean and the deviations of performance variables, Monte Carlo simulation is utilized. In each iteration of optimization, the mean of performance and its variance are calculated based on 300 simulations with S evenly selected within the identified range ΔS.

The results of design variables, linking variables, and total deviations for the Aero subsystem are collected in Table 2. The observation was made that none of these three solutions are the same as the result from the original Aero leader model provided in Table 1, with the difference between the solution of the wing area S being the most significant. Under scenario I the solution of the range (ΔS) is very close to 0 because, under scenario I, the robustness consideration is placed at the highest level. Under the other two scenarios a reasonable range of S (ΔS) is obtained. Compared to scenarios I and II, the results from scenario III represent a good tradeoff between the two aspects of robust design, evidenced by the values of deviation functions included at the end of the table. A smaller value of the deviation function indicates that the goals are better achieved. We observe that the achieved Aero performance is slightly worse under scenarios II and III compared to the one obtained from the model without any robust design considerations (see Table 1). The achieved Aero performance is the worst under scenario I because the robustness consideration is placed at a higher level under this scenario. In Table 2 the resulting deviations of all of the linking variables are listed under Range under each scenario.

Once the leader's model is solved based on the robust design considerations, the solutions of design variables, linking variables, along with the deviations associated with these variables, are passed to the follower model. As shown in Fig. 10, different from the conventional follower model, here the follower Weights has the flexibility of choosing the best value of linking variable S among its range ΔS, as well as the most favorable values of the other linking variables, such as Ld_c, Ld_t, Ld_l, and V_{br} within the deviation ranges as identified by the leader (Table 2).

Results when player Weights is the follower are provided in Table 3. Two of the three scenarios (scenarios II and III) result in

Table 2 Robust solution of Aerodynamics as leader

Variables	Scenario I: level 1 robustness, level 2 performance		Scenario II: level 1 performance, level 2 robustness		Scenario III: robustness and performance at one level	
	Nominal	Range, Δ	Nominal	Range, Δ	Nominal	Range, Δ
B, ft	86.025	——	130.78	——	130.867	——
S, ft^2	1364.17	1.47E−05	1718.64	191.799	1801.18	33.2947
L, ft	144.588	——	114.817	——	113.556	——
			Linking variables			
Ld_L	9.1339	6.48E−04	15.3586	0.1363	15.3187	2.32E−02
Ld_T	6.6426	1.75E−04	12.0694	0.1112	12.0304	1.83E−02
Ld_C	14.7197	1.34E−05	20.2829	0.1883	20.2012	3.23E−02
V_{br}, ft/s	808.344	1.24E−03	687.201	0.6607	686.861	0.146100
	Performance	*Variance*	*Performance*	*Variance*	*Performance*	*Variance*
Deviation functions	0.2599	0.001974	0.24479	231.916	0.24416	39.972

Table 3 Solutions of Weights as follower

Variables	Scenario I	Scenario II	Scenario III
	Weights design variables		
Ti, lbs	48001.3	32132.9	31607.2
Wto, lbs	180320	191097	191318
	Linking variables		
S, ft^2	1364.17	1902.5	1855.99
Ld_L	9.1339	15.3586	15.8193
Ld_T	6.6426	12.1806	12.4784
Ld_C	14.7197	20.4712	20.5827
V_{br}, ft/s	808.344	687.635	679.251
Deviation functions	0.2150	0.19911	0.20163

Table 4 Solution with goal on maximizing the flexibility (scenario IV)

Variables	From leader model		From follower model	
	Nominal	Range, Δ	Nominal	Nominal
B, ft	132.991	——	Ti, lb	31,849.7
S, ft^2	1,769.8	198.875	Wto, lbs	191,706
L, ft	105.174	——	S, ft^2	1,967.07
	Linking variables			
Ld_L	15.575	0.129774	Ld_L	15.575
Ld_t	12.2531	1.05E−01	Ld_tT	12.3829
Ld_c	20.2975	0.190041	Ld_cC	20.4875
V_{br}, ft/s	681.054	1.00063	V_{br}, ft/s	681.913
	Performance	*Variance*	——	*Variance*
Deviation functions	0.2431	47.1	——	0.19806

improved performance for player Weight (from Table 1) with that under scenario II being the most significant. This is reflected in the reduced deviation function values compared to the result without robust design consideration. The most favorable values of linking variables identified by the Weights player (Table 3) are consistent with the tolerable ranges determined by the Aero player (Table 2).

The robust design model for achieving design flexibility is not restricted to the form presented in Fig. 11. Depending on a designer's (leader's) willingness of sacrificing its performance and the need of achieving flexibility, additional constraints and objectives could be included. We illustrate this by considering a case in which the deviations of all of the performance variables are desired to be less than 3% of its nominal value, and ΔS is desired to be as large as possible to provide the most flexibility to the follower. The former consideration is modeled as additional constraints for each performance goal, and the latter is achieved by adding maximizing ΔS as a goal, which is placed at the same level with the goals on performance and robustness. This case is scenario IV, and its results are provided in Table 4. The ΔS, i.e., 198.875 ft^2, identified by the modified leader model is larger than all of the other solutions in Table 2, where the goal of maximizing flexibility is not included. After solving the follower's model using this new range, we observe that the deviation function of the weights system is further reduced to 0.19806. In Table 4 we also list the values of other linking variables identified

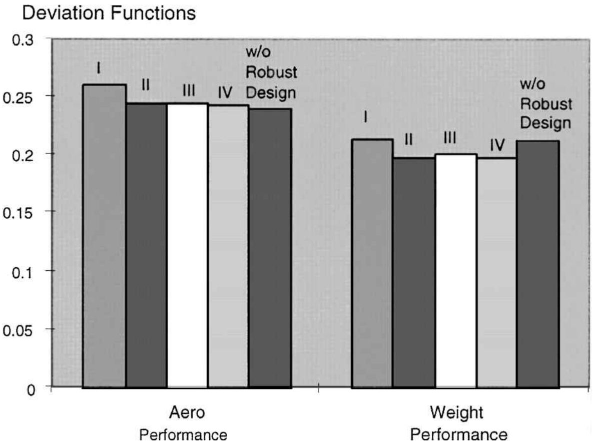

Fig. 11 Comparison of deviation functions.

from the follower model based on the deviation ranges passed from the leader model.

Plots of the deviation functions of the leader's and the follower's performance are provided in Fig. 11 for all of the scenarios exercised. These results are also compared to the solution from the original model without any robust design considerations. Most of the scenarios except scenario I have resulted in improved designs for the player Weights (the lower the deviation function, the better). When maximizing flexibility is added as a goal in the Aero model (scenario IV), this improvement is the most significant. From the comparisons for the Aero performance, the performance of player aerodynamics as the leader has been sacrificed to some extent. The minimum increase occurs in scenarios II, III, and IV, which are all close to 1.58%. Under scenario IV the most significant improvement of the Weights' performance is achieved, being 7.01%. The general profile of Fig. 11 illustrates that the improvement of Weights' performance is obtained at a low cost of sacrificing the needs from Aero. The reason why the results from scenario I are not quite useful is because the emphasis of this model is on achieving the robustness. The objective on achieving mean performance is placed at a lower priority. This arrangement drives the solution toward the lowest flexibility (ΔS is close to 0) in the design region with the minimum performance deviation, but not necessary for a good mean performance. This scenario is therefore not recommended for future applications.

To verify the validity of our results, simulations are conducted to provide the insights into the Aero and Weights models. The grid plots in Figs. 12 and 13 are obtained by exercising the Aero analysis model for different combinations of S and ΔS within their given ranges while fixing the remaining input variables such as B and L according to the solutions of scenario IV. From Fig. 12, to minimize the Aero mean performance deviation function, the mean wing area S is preferred to be around the middle of its range (1200, 2500 ft^2). The range of wing area ΔS has a much smaller impact whereas a smaller value of ΔS is generally preferred.

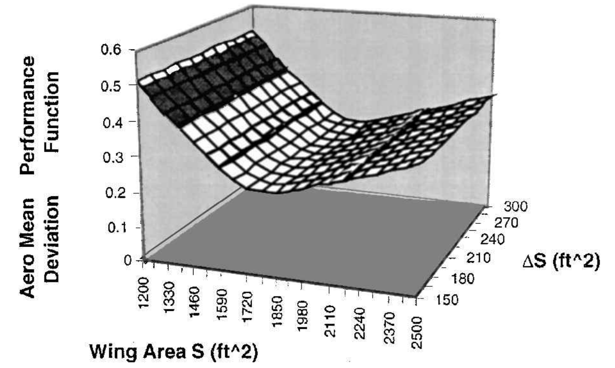

Fig. 12 Grid plots of Aero mean performance deviation function.

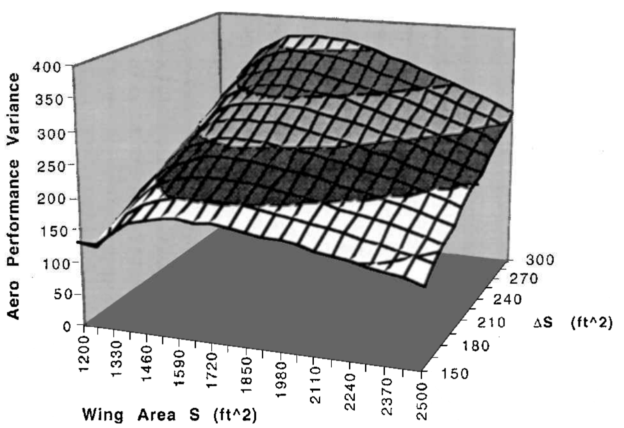

Fig. 13 Grid plots of Aero performance variance.

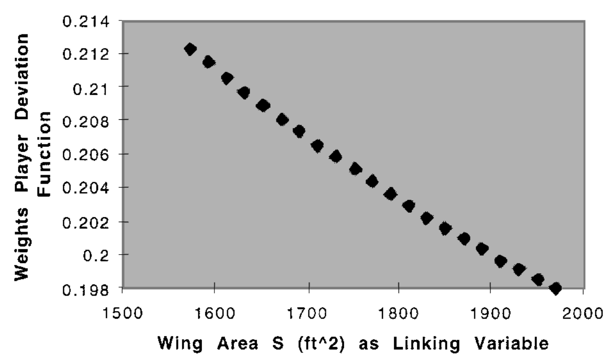

Fig. 14 Impact of linking variable on Weights performance.

This indicates that from the Aero player's point of view smaller deviations of linking variables (less flexibility) are preferred, which is reasonable. Figure 13 further illustrates that a smaller deviation of linking variable (ΔS) results in lower Aero performance variance. Compared to ΔS, S has a much smaller impact on the variance of Aero performance, with those at the upper and lower bounds of S being more favorable. The results obtained under scenario IV, i.e., $S = 1769.9$ and $\Delta S = 198.875$, are consistent with the observed Aero model behavior. This solution illustrates a good tradeoff between the needs of minimizing the Aero mean performance, minimizing the Aero performance deviation, and maximizing the flexibility (ΔS).

The Weights analysis model is also exercised to study the impact of the linking variable on the Weights performance. Figure 14 is obtained by varying the linking variable S within the tolerance range passed from the Aero player while fixing the remainder of the inputs according to those listed under scenario IV (see Table 4). The larger the linking variable S is then the better the Weights' performance, which explains why the value of S determined by the Weights player ($S = 1967.07$ ft^2) sits near the upper bound of the range passed from the Aero player (1769.8 ± 198.875 ft^2). Similar observations can be made for other scenarios.

The Weights leader/Aerodynamics follower scenario is also exercised following a similar procedure. A range of solutions are sought for the linking variables *Wto* and *Ti*, which are the design variables of weights system (leader) that are also the linking variables passed to the Aero system (follower). There is little impact on improving the Aero performance with the flexibility provided. This happens when the linking variables are not the critical factors for improving the follower's performance or because of the restrictions imposed by the constraints of the follower system itself. This problem-dependent insight can be used to predict the influence of disciplines upon each other in order to determine decision-making order, priority, resource allocation, and other design process related parameters.

V. Conclusions

In this paper we developed a design methodology that integrates the robust design concept and the game theoretic approach to multidisciplinary design. Under the Stackelberg leader/follower protocol ranges of solutions are developed for variables that are coupled between multiple players (disciplines). This provides flexibility that helps to resolve the conflicts and disputes of rationality between the interests of multiple disciplines. The robust design concept has been successfully used to search a range of solutions that improve the performance of one discipline as well as to control the performance deviations of the other within a tolerable range. By exercising different scenarios we show that the needs of optimizing performance, minimizing performance deviations, and maximizing flexibility can be modeled at different priority levels in a multiobjective optimization construct.

We acknowledge that there are couplings among disciplines and that we may never be able to eliminate these couplings. However, we are trying to minimize their effects. By minimizing the effects of the decisions made by one discipline upon other disciplines, we feel iteration time can be saved, and the ability to make decisions concurrently can be improved. Our example illustrates the capability of our approach in improving the performance of one discipline while keeping the loss of the other discipline at a minimum. We believe that, by utilizing robust design not only to minimize the effects of external noise factors but to minimize the effects of internal decision factors on multiple disciplines, the benefits of applying multidisciplinary optimization approaches such as CSSO, CO, and game theory to complex design problems can be effectively increased. The principle illustrated in this paper for two disciplines can be extended to multidisciplinary optimization involving more than two players.

Appendix: Rational Reaction Set of Each Discipline

Linking variables y_{12} needed by Weights from Aero [$y_{12} = f(y_{21})$]:

$$S = 1448 + 444.4^* Wto + 175.8^* Ti - 107.5^* Rfr$$
$$- 155.8^* Wto^* Ti - 83.01^* Wto^* Rfr - 83.01^* Ti^* Rfr$$
$$+ 186.5^* Wto^2 + 97.04^* Ti^2 + 15.27^* Rfr^2$$

$$LDc = 18.06 - 1.878^* Wto - 1.380^* Ti + 0.3684^* Rfr$$
$$+ 0.1019^* Wto^* Ti + 0.19^* Wto^* Rfr + 0.19^* Ti^* Rfr$$
$$- 0.4238^* Wto^2 + 0.1319^* Ti^2 - 0.685^* Rfr^2$$

$$Vbr = 744.9 + 6.421^* Wto - 52.37^* Ti + 7.532^* Rfr$$
$$+ 15.79^* Wto^* Ti + 6.924^* Wto^* Ti + 6.924^* Ti^* Rfr$$
$$- 31.64^* Wto^2 - 9.419^* Ti^2 + 7.828^* Rfr^2$$

$$LDl = 14.12 + 1.47^* Wto - 2.142^* Ti + 2.601^* Rfr$$
$$- 0.2179^* Wto^* Ti + 0.2556^* Wto^* Rfr + 0.0879^* Ti^* Rfr$$
$$+ 0.0998^* Wto^2 + 0.4169^* Ti^2 - 0.6684^* Rfr^2$$

$$LDt = 9.698 - 0.9576^* Wto - 1.97^* Ti + 0.04071^* Rfr - 0.336^* Wto^* Ti - 0.05205^* Wto^* Rfr - 0.05205^* Ti^* Rfr + 0.1815^* Wto^2 + 0.7128^* Ti^2 - 0.899^* Rfr^2$$

Linking variables y_{21} needed by Aero from Weights $[y_{21} = f(y_{12})]$:

$$Wto = 216000 + 15040^* S - 11300^* Vbr - 163.4^* LDl - 5318^* LDc + 20930^* LDt - 305.0^* S^* Vbr - 148.6^* S^* LDl + 2694^* S^* LDc + 13580^* S^* LDt - 158.9^* Vbr^* LDl + 2512^* Vbr^* LDc - 9457^* Vbr^* LDt + 425.8^* LDl^* LDc - 173.6^* LDl^* LDt - 3912^* LDc^* LDt - 22370^* S^2 - 1624^* Vbr^2 + 11730^* LDl^2 + 2571^* LDc^2 - 24730^* LDt^2$$

$$Ti = 39120 - 284.1^* S - 2565^* Vbr - 547.2^* LDl - 1558^* LDc - 10170^* LDt + 1680^* S^* Vbr - 559.6^* S^* LDl + 1525^* S^* LDc - 784.5^* S^* LDt - 447.1^* Vbr^* LDl + 1409^* Vbr^* LDc - 2194^* Vbr^* LDt - 149^* LDl^* LDc - 581.4^* LDl^* LDt - 1355^* LDc^* LDt - 4058^* S^2 - 229.7^* Vbr^2 + 2212^* LDl^2 + 988.2^* LDc^2 + 6190^* LDt^2$$

$$Rfr = 0.3145 - 0.0833^* Vbr - 0.06146^* LDc + 0.01412^* Vbr^* LDc + 0.00003624^* S^2 + 0.02323^* Vbr^2 + 0.00003624^* LDl^2 + 0.01261^* LDc^2 + 0.00003624^* LDt^2$$

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